DAMPENING VARIABILITY BY USING SMOOTHING REPLENISHMENT RULES

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A major cause of supply chain deficiencies is the bullwhip effect which can be substantial even over a single echelon. This effect refers to the tendency of the variance of the replenishment orders to increase as it moves up a supply chain. Supply chain managers experience this variance amplification in both inventory levels and replenishment orders. As a result, companies face shortages or bloated inventories, run-away transportation and warehousing costs and major production adjustment costs. In this article we analyse a major cause of the bullwhip effect and suggest a remedy. We focus on a smoothing replenishment rule that is able to reduce the bullwhip effect across a single echelon. In general, dampening variability in orders may have a negative impact on customer service due to inventory variance increases. We therefore quantify the variance of the net stock and compute the required safety stock as a function of the smoothing required. Our analysis shows that bullwhip can be satisfactorily managed without unduly increasing stock levels to maintain target fill rates.
(BULLWHIP EFFECT; SUPPLY CHAIN MANAGEMENT; INVENTORY MANAGEMENT; VARIANCE REDUCTION)

1. Introduction

There is ample anecdotic evidence that many companies experience significant extra costs due to supply chain problems. Konicki (2002) reports on a major retailer's inability to master supply chain logistical problems. The company faced sharp drops in demand for products and sales merchandise was often out of stock when customers got to the store. Furthermore, bloated stocks sat alongside these empty racks and display shelves, but they were no guarantee of high customer service levels. It is a formidable job for logistics managers to design order management systems that optimally match pipelines to the marketplace (see Childerhouse, Aitken and Towill (2002) and Christopher and Towill (2002)).

What is causing all this trouble? Why is it that the material flow is so hard to predict in supply networks? There are, for sure, many causes of these deficiencies. In this paper however we will focus on the bullwhip problem. The bullwhip problem refers to the tendency of replenishment orders to increase in variability as it moves up a supply chain. As smooth final customer demand patterns are transformed into highly erratic demand patterns for suppliers; the information in the chain gets distorted. The bullwhip is characterised by oscillations of orders at each level of the supply chain and an amplification of these oscillations farther up the chain away from the marketplace (Croson and Donohue (2003)). Jay Forrester (1961) was among the first researchers to describe this phenomenon, then called "Demand Amplification". The Beer Game developed at MIT is very popular in many business schools and executive seminars and is very useful for illustrating the bullwhip problem, Sterman (1989).

Procter and Gamble first coined the phrase "bullwhip effect" to describe the ordering behaviour witnessed between customers and suppliers of *Pampers* diapers. While diapers enjoy a fairly constant consumption rate, P&G found that wholesale orders tended to fluctuate considerably over time. They observed a further amplification of the oscillations of orders placed to their suppliers of raw material. The bullwhip problem has been given a lot of academic attention after the important contribution of Lee et al. (1997).

There is also a lot of empirical evidence of bullwhip. Our own data shows that the coefficient of variation (the ratio of the standard deviation over the mean) of weekly retail sales typically range between 0.15 and 0.50 whereas the coefficient of variation of production orders (even in small batch driven environments) is typically in the range of 2 to 3. Moreover, the bullwhip effect is multiplicative in traditional supply chains. There is ample evidence in many business environments to verify this and mathematical models will prove it (Dejonckheere, Disney, Lambrecht and Towill (2004)). One of the principal reasons used to justify investments in inventories is its role as a buffer as it is believed that inventories have a stabilising effect on material replenishment. Counterintuitively, however inventory management policies can have a destabilising effect by increasing the volatility of demand in the supply chain (Baganha and Cohen, 1998).

We will now review causes of the bullwhip effect as available in the literature, and investigate ways to alleviate and to overcome the problem. We distinguish between operational and behavioural causes. The behavioural causes are rather straightforward. Supply chain managers may not always be completely rational. Managers over-react (or under-react) to demand changes. People often try to read “too much signal” into a series of demand history as it changes over time. Decision makers sometimes over-react to trade and newspaper reports, customer complaints and anecdotes of negative customer reactions. Moreover, there are cognitive limitations as supply chain networks are often very complicated, operating in a highly uncertain environment with limited access to data. Croson and Donohue (2002) and Sterman (1989) found that decision makers consistently under-weight the supply chain. This means that they don’t have a clear idea of what will be available from the order pipeline. This data masking induces some form of decision bias. Strategies to alleviate this problem include: sharing Point-Of-Sales data, sharing inventory and demand information, centralizing ordering decisions and using formal forecasting techniques correctly (we will come back on this issue later on in this paper). Although sharing of information is often cited as a bullwhip remedy, there is evidence that retailers and suppliers are often not willing to share information (see de Treville, Shapiro and Hameri

(2004)). Moreover, it is not clear whether POS information is unambiguously beneficial (Steckel, Gupta and Banerji (2004)).

2. Known causes of the bullwhip effect.

Lee et al (1997) identify five major operational causes of the bullwhip: demand signal processing, lead-time, order batching, price fluctuations and rationing and shortage gaming. We understand demand signal processing as the practice of decision makers adjusting the parameters of the inventory replenishment rule. Target stock levels, safety stocks and demand forecasts are updated in face of new information or deviations from targets. These “rational” adjustments create erratic responses. We will also show that it is possible to design replenishment rules that have a stabilizing, smoothing effect on orders. It is important to realize that most players in supply chains do not respond directly to the market but respond to replenishment demand from downstream echelons. This is why local optimisation often results in global disharmony. It is therefore claimed that centralized control (e.g. Distribution Requirements Planning, Vendor Managed Inventories) is superior to decentralized control (disconnected supply chains).

A second major cause of the bullwhip problem is the lead-time. Lead-times are made of two components; the physical delays as well as the information delays. The lead-time is a key parameter for calculating safety stock, reorder points and order-up-to levels. The increase in variability is magnified with increasing lead-time. A way to alleviate this problem is lead-time compression. The information delay can be reduced by better communication technologies (web-enabled communication, EDI, e-procurement etc) and the order fulfilment lead-time (the physical lead-time) can be reduced by investment in production technology and process, strategic supplier partnerships (supplier hubs, logistics integrators etc) or by eliminating channel intermediaries (direct channels, ‘the Dell model’).

A third well-known bullwhip creator is the practice of order batching. Economies of scale in ordering, production set-ups or transportation will quite clearly increase order variability (Burbidge (1989)). Reduction of set-up, ordering and handling costs is of course a way to alleviate this problem.

The fourth major cause of bullwhip as highlighted by Lee et al (1997) has to do with price fluctuations. Retailers often offer price discounts, quantity discounts, coupons or in-store promotions. This results in forward buying where retailers (as well as consumers) buy in advance and in quantities that do not reflect their immediate needs. Pricing strategies (ranging from deep promotions to Every Day Low Price) should clearly be connected to supply and replenishment policies. However, it is not sure from a marketing perspective whether the positive supply chain effect (e.g. higher efficiencies) outweighs the potential negative marketing effect (e.g. demand-depressing side effects). We refer to Ortmeyer et al. (1991) and Butman (2002) for more details on issues in this operations management, marketing interface.

In general, it is important to transmit into the supply chain the correct demand information. An accurate forecast (see Chen, Drezner, Ryan and Simchi-Levi (2000)) will assist the upstream suppliers' capacity and material planning. We may want to stimulate forecast accuracy and to penalise forecast errors. We may want to limit the ability to revise forecasts over time, or we may negotiate flexibility contracts with customers (based on risk sharing). These are all ways to have demand better under control and to view forecasting as more than just a courtesy call.

A further cause of the bullwhip has to do with rationing and shortage gaming. Inflated orders placed by supply chain members during shortage periods tend to magnify the bullwhip effect. Such orders are common when retailers and distributors suspect that a product will be in short supply. Exaggerated customers orders make it hard for manufacturers to forecast the real demand level. A very simple countermeasure is to allocate products proportional to sales in previous periods and not proportional to what has been ordered.

The well known Lee et al. classification scheme described above can be complemented by other classification schemes. One such interesting scheme relates to contract structures in supply chains. Coordination and cooperative behavior in the supply network can be achieved by appropriate buyer-supply contracts. Tsay, Nahmias and Agrawal (1999) offer an excellent survey of the contract literature. The (re)allocation of decision rights is one example (e.g. retailer versus vendor managed inventory, allocation of inventory in a distribution network). Other examples are pricing decisions (discount schedules, joint lot-sizing ,…), minimum (or maximum) purchase commitments (including order timing), quantity flexibility (risk sharing, backup agreements,…), buyback or return clauses, allocation rules when multiple retailers compete for a limited supply of products, lead times (the economic value of lead time reduction) and the division of coordination benefits among the supply chain parties. All of the above measures may improve the overall supply chain profitability. The coordination improves, the information distortion is reduced and consequently the contractual arrangements may reduce the variability in the material flow.

This short overview of the causes of the bullwhip effect (and a short summary of potential remedies and contractual implications) highlights that the bullwhip effect is a very complex issue. It touches on all aspects of supply chain management. In this article, we will limit ourselves to one specific cause, the (ab)use of replenishment rules. We propose an order smoothing rule that is able to dampen variability. In section 3 we introduce the order-up-to replenishment rule and demonstrate that it creates bullwhip. In section 4 we introduce a new smoothing replenishment rule. In section 5 we focus on the link between replenishment rules and customer service for the matched controller case. In section 6 we analyze the smoothing rule with unmatched controllers. The major contribution of this paper is the study of the link between bullwhip reduction (order smoothing) and the impact on inventory fluctuations (and consequently customer service). Section 7 concludes this paper.

3. The Order-Up-To replenishment rule

There are many different types of replenishment policies (for example see Zipkin (2000) and Silver, Pyke and Peterson (1998)) The two most commonly used are: the periodic review, replenishment interval, Order-Up-To (OUT) policy and the continuous review, reorder point, order quantity model. Given the common practice in retailing to replenish inventories frequently (daily, weekly, monthly) and the tendency of manufacturers to produce to demand, we will focus our analysis on the replenishment strategies known as Order-Up-To (OUT) policies. In such a system we track the inventory position (= amount on-hand + inventory on-order – backlog). The inventory position is reviewed every period (e.g. daily, weekly) and an order is placed to raise the inventory position up to an order-up-to or base stock level that determines order quantities. This policy is sometimes preferred due to qualitative benefits of following a regular repeating schedule of inventory replenishment. Both the review period and the order-up-to level are decision variables but in order to simplify the analysis we set the review period equal to one base period (day, week or month). The OUT level equals the expected demand during the risk period and a safety stock to cover higher than expected demands during the same risk period. The risk period equals the physical lead-time (*Tp* periods) and the review period (1 period). Consequently,

$$S_t = \hat{D}^{Tp+1} + k.\hat{\sigma}^{Tp+1}.$$

S_t is the OUT level used in period t and $\hat{D}^{Tp+1}$ is an estimate of mean demand over Tp+1 periods (we assume $\hat{D}^{Tp+1} = (Tp+1)\hat{D}^{\alpha}{}_t$, where $\hat{D}^{\alpha}{}_t$ is the estimate of demand in the next period calculated e.g. with exponential smoothing, with a smoothing constant α). $\hat{\sigma}^{Tp+1}$ is a (constant) estimate of the standard deviation of the forecast error over Tp+1 periods. k is a constant chosen to meet a desired service level. In this analysis we opt for Fill Rate as the measure of customer service. To simplify the analysis we replace the safety stock term by $a.\hat{D}^{\alpha}{}_{t}$ (this can always be done and it makes the analysis somewhat easier). After this substitution we obtain,

$$S_t = (Tp + 1 + a)\hat{D}^{\alpha}{}_{t} \,. \tag{1}$$

This more general form of the OUT policy defines the risk period as *Tp*+1+*a* and consequently immediately includes the safety stock.

Suppose that the demand process is normally, independently and identically distributed (i.i.d.) over time, then it is quite clear that the best estimate of all future demands is simply the long-term average demand, $\overline{D}$. Formula (1) then becomes,

$$S = (Tp + 1 + a)\overline{D} \,. \tag{2}$$

What happens now if we apply the above replenishment rule (2) (using $\overline{D}$ as an estimator). The answer to that question is simple and known to most inventory managers (see Dejonckheere et al. (2003)). The OUT policy is generating replenishment orders that are the same as the last periods observed demand. We simply order what the demand was in the base period (similar to a Just-In-Time strategy), that’s why this policy is also called “passing on orders” or “lot for lot” or even sometimes “continuous replenishment” when the length of the planning period has been shortened. Either way, the variability of the replenishment orders is exactly the same as the variability of the original demand. So why do we observe variance amplification in the real world? The answer to that key question is that decision makers do not know the demand (over the lead-time) and consequently they have to rely on demand forecasts. Moreover, they adjust the OUT levels for seemingly “intuitively correct” reasons. Because of the fact that the true underlying distribution of demand is not directly observed (only the actual demand values are observed) many inventory theory researchers suggest the use of adaptive inventory control mechanisms (see e.g. Treharne and Sox (2002)). One can show that adaptive policies provide better solutions than the certainty equivalent control policies commonly used in practice. Unfortunately these adjustments create bullwhip. This observation was already well described by Forrester (1961) and was very elegantly proved by Chen et al (2000).

Let us illustrate one possible adjustment strategy. Assume that the decision-maker follows an OUT policy, that means the retailer orders what the demand was, but we adjust this quantity by the difference between the target safety stock ($a.\overline{D}_t$) and the actual physical inventory at the end of the period. This is a quite logical adjustment, if the physical inventory at the end of a period is less than the safety stock, order more and vice versa. This rather logical, and at first sight innocent adjustment rule, has a very devastating effect on the bullwhip as is illustrated in Figure 1. The example is introduced in the next paragraph.

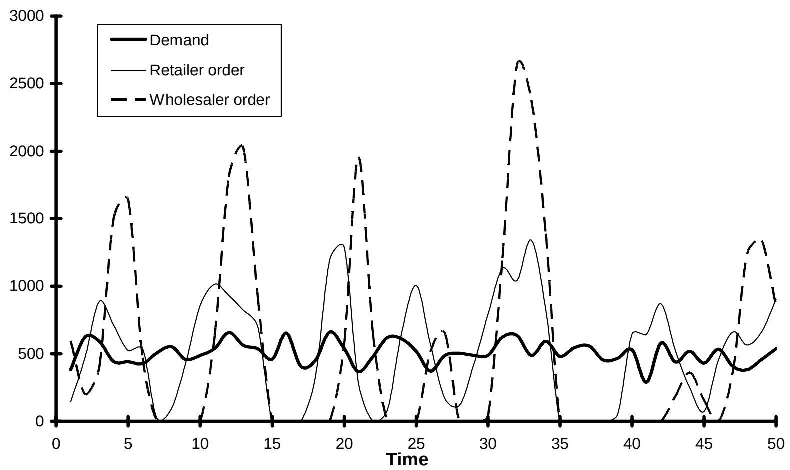

FIGURE 1. The impact of adjustments by various “players” in the supply chain

Assume a supply chain consisting of customers, a retailer and a wholesaler. The retailer physical lead-time equals two periods and the wholesaler lead-time also equals two periods. Further, assume a normally distributed demand process with an expected value of 500 ($\overline{D} = 500$) units per period and a standard deviation of 100 units (the coefficient of variation equals 0.2). Furthermore set the safety stock, a = 0.5. The OUT level for the retailer equals (2+1+0.5)500=1750 and the OUT level for the wholesaler equals (2+1+0.5)500=1750. The safety stock equals 500(0.5)=250 units. An OUT policy results in a replenishment pattern with a variance equal to the variance of the demand pattern. The adjustment policy explained in the previous paragraph, however, results in the replenishment patterns shown in figure 1. No need to say that there is a very significant variance amplification effect. The so-called "adaptive" inventory policies may have a devastating effect on the amplification of oscillations.

Unfortunately, the use of forecasting tools has exactly the same impact. Suppose we use exponential smoothing as a forecasting tool:

$$\hat{D}_t = \hat{D}_{t-1} + \alpha(D_t - \hat{D}_{t-1}) \qquad (3)$$

Suppose that the demand was 100 units for the last ten periods, and that in period eleven the demand increases to 150 units. Assume 4 stages in the supply chain each with a four period lead-time (Tp=2, review period = 1 and a = 1)). The first member of the chain will forecast a demand of 110 units (use α=0.2) and its OUT level equals 440. Given that the demand increased to 150 units, the inventory position will decrease (250 units). The order quantity will consequently increase, in this example from 100 to 190 units. This order quantity is now transferred to the next link in the chain and exactly the same will happen there, that means, the OUT level increases and the inventory position decreases. After 4 stages the order quantity equals 626 units whereas it was only 100 units in the first ten periods. That's how the bullwhip effect works. The key driver herein is the "full adjustment policy" used to recover inventory errors (this example is based on a class note example of Ananth V. Iyer (private communication)).

The bullwhip problem is akin to a common situation we face every morning. As we stand underneath a cold shower and turn the hot tap too quickly, the water, a few moments later (lead-time), becomes too hot and we respond by reaching for the cold tap or turning back the hot tap. These "full" adjustments are undesirable. We all know that, when in the shower, we should turn the taps very slowly in order to get the temperature "just right". Well, the same issue is prevalent in a supply chain, we must turn the taps very slowly also. The key word here is "fractional adjustment" and this is well known to control engineers. The consequential smoothing replenishment strategy is the subject of the next section.

4. A smoothing replenishment rule: smooth is smart

Smoothing is a well known method to reduce variability. A number of (single product) production level smoothing rules were developed in the 1950's. Vassian (1955) developed a periodic review reorder rule resulting in a minimum inventory variance to any sequence of forecasting errors. Also Magee (1956 and 1958) did pioneering work on the development of production level smoothing rules. Our results confirm and extend the work of Magee. We also refer to the work of Simon (1952) using servomechanism theory to study the dynamics of smoothing rules. Another key paper was written by Deziel and Eilon (1967), and our research outcomes confirm and extend their work.

The more recent work on smoothing replenishment rules can be found in Dejonckheere, Disney, Lambrecht and Towill (2003 and 2004) and Balakrishnan, Geunes and Pangburn (2004)

It is also very instructive to review the literature on what is known as the Holt, Modigliani, Muth and Simon (1960) decision rules. The problem faced was simply the determination on a period by period basis of both a production quantity and an employment level under quadratic cost functions. This resulted in two smoothing rules and formed the basis of numerous extensions. One of these extensions was Bowmans' Management Coefficient Theory (1963). This theory has an interesting insight very useful for our bullwhip reduction research. Bowman claims that bias (deviating from optimal behaviour) causes a relatively small criteria loss, whereas variance (due to frequent adjustments) will cause large criteria losses. In other words, it is variance in the decision making rather than bias that hurts. The "modern" version of this problem can be found in control of supply chain variability.

Let's go back to formula (1) and decompose it as follows,

$$O_t = S_t - \text{inventory position}$$

Where O_t is the ordering decision made at the end of period *t*. The inventory position equals the net stock (*NS*) plus inventory on order (Work In Progress or WIP). The net stock equals inventory at hand minus backlog.

$$O_t = (Tp + 1 + a)\hat{D}_t^{\alpha} - NS_t - WIP_t$$

$$O_t = \hat{D}^{\alpha}{}_t + (a\hat{D}_t^{\alpha} - NS_t) + (Tp.\hat{D}_t^{\alpha} - WIP_t) \qquad (4)$$

where $a\hat{D}^{\alpha}{}_t$ can be viewed as a target net stock (safety stock) and $Tp.\hat{D}^{\alpha}{}_t$ as a target pipeline stock (on order inventory). Expression (4) is the same as expression (1), but we decomposed the original formula into three components: a demand forecast, a net stock discrepancy term and a WIP or pipeline discrepancy term, see Dejonckheere, Disney, Lambrecht and Towill (2003). Moreover, if we now want to turn the taps slowly, we can give an appropriate weight to the discrepancies as in expression (5).

$$O_t = \hat{D}^{\alpha}{}_t + \beta(a\hat{D}_t^{\alpha} - NS_t) + \gamma(Tp.\hat{D}_t^{\alpha} - WIP_t) \qquad (5)$$

We now have three controllers α, β and γ that will enable us to tune the dynamic behaviour of the supply chain. For $\beta = \gamma > 1$ bullwhip is created (amplified) and for $\beta = \gamma < 1$ we will create a smoothed replenishment pattern (dampening).

This is illustrated in figure 2. We take the same example as before (i.i.d. normal distribution with $\overline{D}$ = 500 and $\sigma_D = 100$ and *Tp*=2) and we use $\overline{D}$ as an estimator; furthermore for simplicity, we assume for this example that $\beta = \gamma$.

The first controller α is simply the smoothing constant in the exponential smoothing forecasting rule. Smaller values will produce smoother responses; larger values create more bullwhip. Note that in this paper we have set α=0, to match the assumption that demand is a stationary i.i.d. random variable, simplifying the equations presented. When demand is not stationary, that is, there is a genuine change in mean demand or some auto-correlation in the demand signal exists, then 0<α<2 may be better suited (or indeed a different forecasting mechanism). The major advantage of this modification to the OUT policy is that it filters out "noise" in the marketplace sales (through the dampened feedback), whilst tracking genuine changes in demand (admittedly with a lag). By doing this, companies can avoid excess costs due to unnecessary ramping up and down production or ordering levels. The optimal values of the three controllers are obviously sensitive to the economics of the supply chain in question.

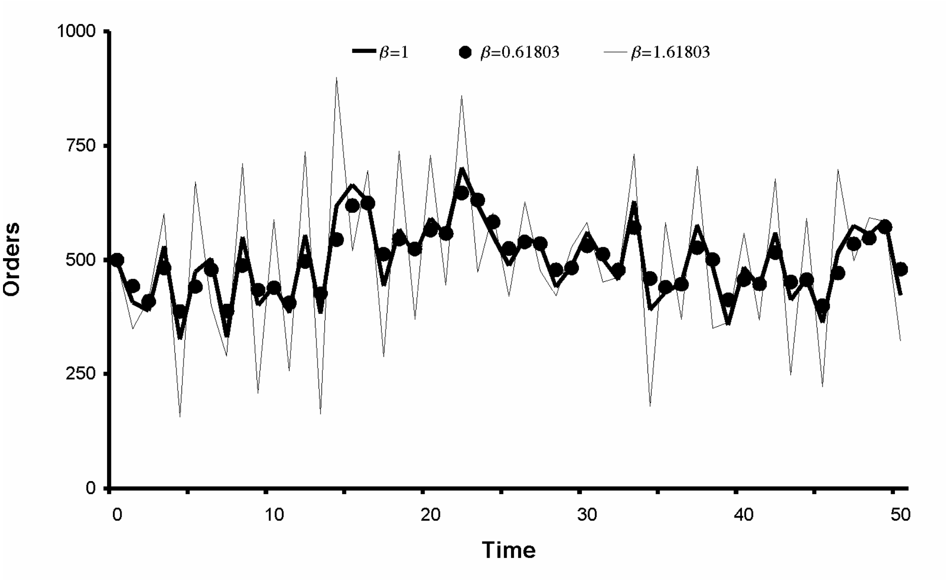

FIGURE 2. A smoothing replenishment rule

5. The smoothing rule under stationary demand and matched controllers

Under stationary demand we set $\hat{D}^{\alpha}{}_{t} = \overline{D}$ and $\beta = \gamma$ (matched feedback controllers) and equation (5) reduces to :

$$O_t = \overline{D} + \beta(a\overline{D} - NS_t + Tp.\overline{D} - WIP_t)$$

We next determine O_{t-1}

$$O_{t-1} = \overline{D} + \beta(a\overline{D} - NS_{t-1} + Tp.\overline{D} - WIP_{t-1})$$

It can easily be seen that:

$$NS_t = NS_{t-1} + O_{t-(Tp+1)} - D_t$$

and

$$WIP_t - WIP_{t-1} = O_{t-(Tp+1)} - O_{t-1}$$

After substitution we obtain:

$$O_t - O_{t-1} = \beta(D_t - O_{t-1})$$

or

$$O_t = O_{t-1} + \beta(D_t - O_{t-1}) \text{ or } O_t = (1-\beta)O_{t-1} + \beta D_t \qquad (6)$$

Formula (6) is equal to the exponential smoothing formula (3) where $\hat{D}_t$ has been replaced by O_t. If $\beta = 1$ expression (6) reduces to $O_t = D_t$. This is equivalent to the unadjusted order-up-to policy.

Expanding equation (6) results in:

$$O_t = \beta D_t + \beta(1-\beta)D_{t-1} + \beta(1-\beta)^2 D_{t-2} + ... + (1-\beta)^n O_{t-n} \qquad (7)$$

Equation (6) and (7) tells us that the order-up-to policy reduces to exponential smoothing on replenishment orders or the order quantity equals a convex combination of previous demand realizations. Balakrishnan, Geunes and Pangburn (2004) propose a general (linear) order smoothing policy of the following form

$$O_t = \sum_{k=0}^{\infty} \alpha_k D_{t-k}$$

Our smoothing policy is clearly a special case of the above general smoothing rule, more specifically we propose an exponential smoothing scheme for the smoothing coefficients α_k.

No need to say that policy (6) will automatically yield less upstream variance than an unadjusted order-up-to policy.

From (6) we can easily deduct that the autocorrelation between O_t and O_{t-x}=$(1-\beta)^x$. This implies that smoothing (β<1) generates a positively correlated order stream. We now have to find expressions for two important metrics, first the bullwhip (defined as the ratio of the variance of the orders over the variance of demand) and next the net stock (physical end of period inventory) variance amplification (defined as the ratio of the variance of the net stock over the variance of demand). As will be discussed later, both measures are of crucial importance. The situation is graphically represented in figure 3.

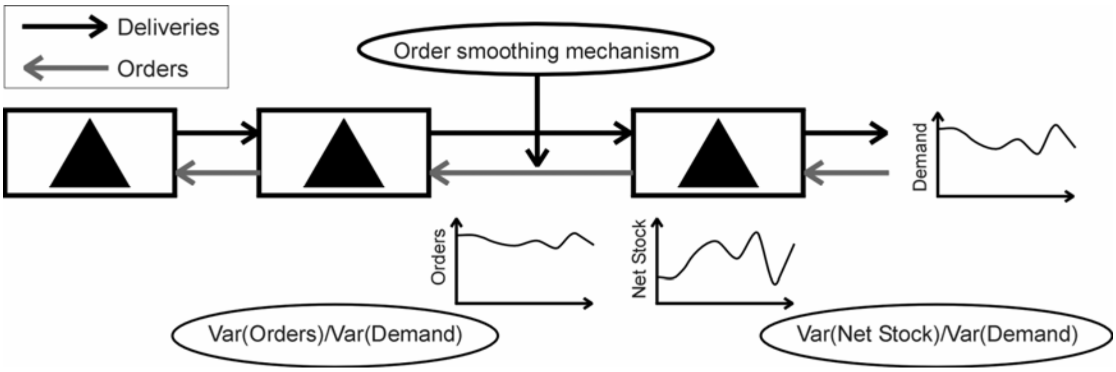

FIGURE 3. The two key metrics related to replenishment policies

We first define a measure of the bullwhip effect. The variance of orders can be found through statistical analysis or by applying control systems engineering methodology (z-transforms, see Dejonckheere et al. (2003)). The proofs of all the formulas presented in this paper are available from the authors.

$$Bullwhip = \frac{\sigma_O^2}{\sigma_D^2} = \frac{\beta}{2-\beta} \qquad (8)$$

or graphically:

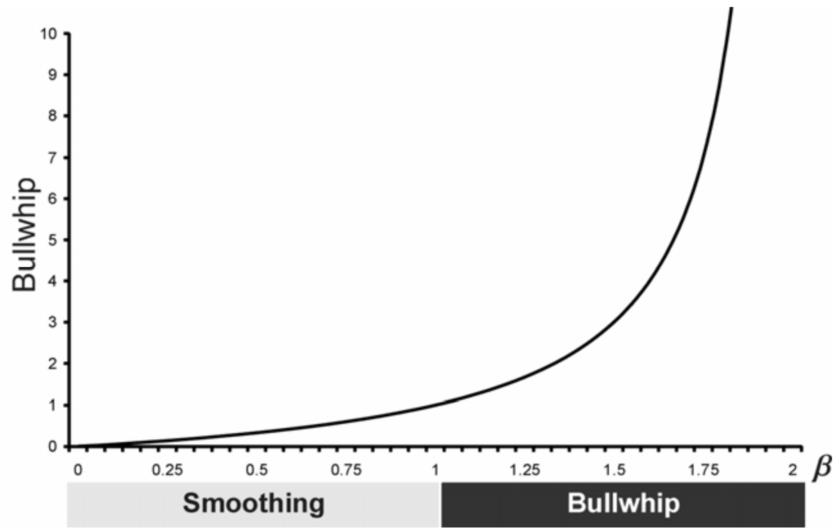

FIGURE 4. Bullwhip with stationary demand and matched feedback controllers

We observe that for the case of stationary demand and $\gamma = \beta$, bullwhip is independent of lead-time.

So far we have been concentrating on the variance of orders placed. This is, however, only one side of the coin. We should also study the variance of inventory, because that variance will have an immediate effect on customer service: the higher the variance, the more stock will be needed to maintain customer service at the target level. Another argument why the variance of inventory is important goes as follows: in a production environment with a wide product range, the bullwhip effect, measured at the item level, may be dampened overall (at the manufacturing level) because of the portfolio effect, but this will not be the case for fill rate considerations. Shortages will generally not be compensated by excess inventory of a different item.

Recall that 'net stock' refers to NS_t in (4)

Remember also that $\beta = \gamma = 1$ results in a bullwhip measure of 1 as we have a pure chase policy. In such a case the inventory fluctuations will be minimal.

Intuitively, we expect smooth ordering patterns ($\beta = \gamma < 1$) will result in higher inventory fluctuations and consequently in a poorer fill rate, and this is indeed the case. Defining a measure of net stock variance amplification as,

$$NSAmp = \frac{\sigma_{NS}^2}{\sigma_D^2} \,. \qquad (9)$$

Statistical analysis or a control systems engineering methodology will result in the following interesting expressions for *NSAmp*.

$$NSAmp = 1 + Tp + \frac{(1-\beta)^2}{(2-\beta)\beta} \qquad (10)$$

NSAmp clearly has a 'review and physical lead time' component and a 'smoothing' component. Figure 5 shows NSAmp as a function of $\gamma = \beta$ for Tp = 2. For a pure OUT ($\beta = 1$) strategy, the smoothing component equals zero. Note that even then, inventory variance exceeds demand variance by a factor 3 (= 1+*Tp*). Otherwise, for 0<$\beta = \gamma < 1$, the smoothing component is always positive. As expected, smooth replenishments increase the variance of inventory.

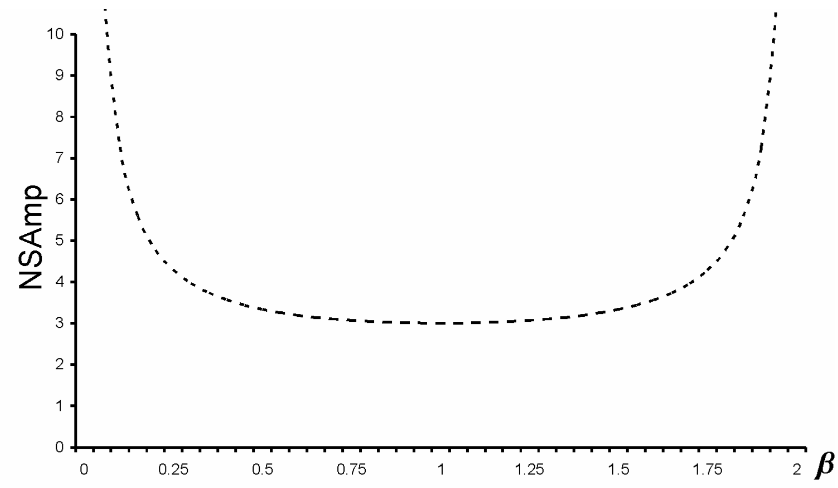

FIGURE 5. NSAmp as a function of $\beta = \gamma$; $Tp = 2$

Figure 5 shows that *NSAmp*

- is minimal at $\beta = 1$;

- the longer the lead time Tp, the smaller the relative impact of smoothing;
- increases with decreasing β, but also with increasing β. This means, that from an inventory point of view, smoothing ($\beta < 1$) and bullwhip ($\beta > 1$) are equally as 'bad'.

These observations lead to an interesting trade-off between bullwhip avoidance and customer service.

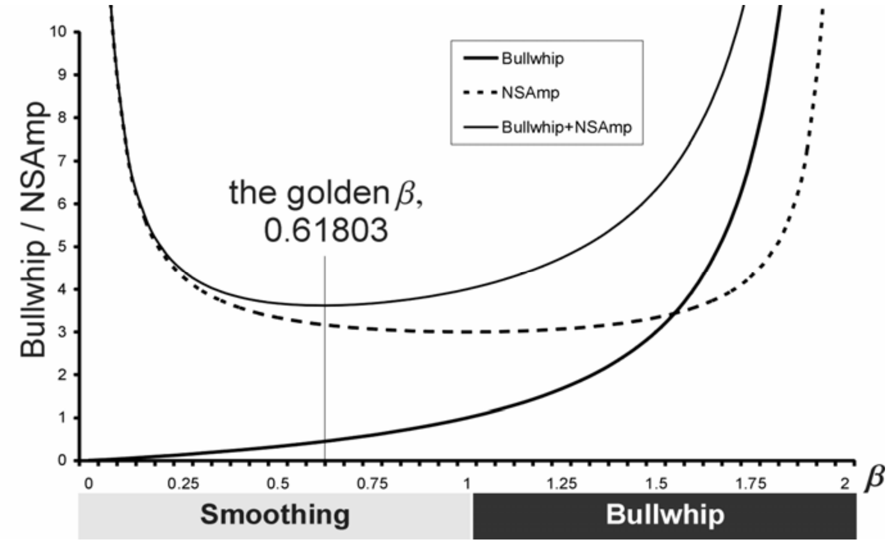

FIGURE 6. The variability trade-off (using the same data as Figures 4 and 5)

It can be shown that the sum of NSAmp and Bullwhip is minimised at β = 0.618, irrespective of lead-time. As a side note: 0.618, and its inverse, 1.618, is known since ancient history as the Golden Ratio, often found in many forms of the arts and nature. For example it describes the optimal placement of seeds and leaves in growing plants, the optimal ratio of female and male bees and geometric patterns in architecture.

By adding bullwhip and net stock amplification we assume that inventory holding and shortage costs are linearly related to the net stock variance and that flexibility costs resulting from unstable schedules are linearly related to order variance (bullwhip). Furthermore we assume that inventory variance is equally as costly as order variance. In a business application, it is perfectly possible that the bullwhip effect and net stock amplification are not equally important. In this case we have to apply weights to these factors, this in turn may change the shape of the curve in Figure 6. It is also interesting to note that the costs associated with the bullwhip effect, have a greater impact on upstream supply chain parties (e.g. a manufacturer) whereas the costs associated with the net stock amplification will mainly impact the downstream echelon (e.g. the retailer). Solving the bullwhip problem consequently asks for coordinated actions throughout the chain.

5.1. *Impact of the smoothing rule on fill rate (customer service) under stationary demand and matched controllers*

Net Stock Variance (let alone variance amplification) is not a common supply chain measure. However, we need it to calculate fill rate, a popular customer service measure (Zipkin, 2000). Fill rate is defined as the fraction of volume delivered from inventory.

$$\text{Fill Rate} = 1 - \frac{\text{ESPRC}}{\text{expected demand per replenishment cycle}} \tag{11}$$

with ESPRC the expected shortages per replenishment cycle. Since we place an order every period, ESPRC can be simplified to ESPP, the expected shortages per period.

The classical formulas available to calculate the fill rate for periodic review replenishment rules assume a fixed OUT level. However, in general, the very nature of our smoothing rule will have a variable OUT level. Therefore, we need to approach the fill rate calculation in a different way.

In a given period t, there will be a shortage if NS_t, the net stock at the end of the period is negative. The size of the shortage is equal to $-NS_t$. Therefore, we can write ESPP as follows:

$$ESPP = E\left(-NS_t \left| NS_t < 0\right.\right) \tag{12}$$

Since NS_t is a linear combination of normal random variables, NS_t has a normal distribution, with average TNS, the target net stock, and standard deviation σ_{NS}:

$$\sigma_{NS} = \sigma_D \sqrt{1 + Tp + \frac{(1-\beta)^2}{(2-\beta)\beta}} \tag{13}$$

Building on the analogy between the typical equations that need to be solved to determine fill rates in classical replenishment rules and equation (12), we write:

$$ESPP = L(z) \times \sigma_{NS} \tag{14}$$

and

$$TNS = z \times \sigma_{NS}\,. \tag{15}$$

For any given Target Net Stock level, the safety factor z can be easily calculated using (15). Using tables of the standard normal distribution, we can determine *L(z)* (the standard normal loss function). Finally, with (14) and (11), we can calculate the fill rate associated with the given TNS level.

It is equally straightforward to determine the required TNS level to reach a target fill rate: first we need to calculate *L(z)* using (14). With the tables of the standard normal distribution and the loss function, we can then determine z and finally TNS with equation (15).

While we approached the calculation of TNS and fill rate in an alternative way, the reader will notice that our method yields exactly the same result as the classical OR textbook methods if we consider β=1 (no smoothing). In this case, equation (15) simplifies to:

$$TNS = z \times \sigma_D \times \sqrt{1 + Tp}$$

a well known formula used in many inventory models. The fraction in expression (13) can be interpreted as the extra time a unit spends in inventory due to smoothing or bullwhip creation.

The TNS of equation (15) can also be expressed as a number of periods coverage, a:

$$TNS = a \times \overline{D} \tag{16}$$

While the safety factor z is related to σ_{NS}, a , represents how many periods of average demand $\overline{D}$ are covered by the Target Net Stock (TNS). The resulting 'smoothing' replenishment rule, guaranteeing a specified fill rate equals:

$$O_t = \overline{D} + \beta\ ((Tp+a)\ \overline{D} - NS_t - WIP_t) \tag{17}$$

In order to quantify the trade-off between the degree of 'smoothing' and the associated investment in safety stock we have to know the costs involved. Our experience is that a lot of 'smoothing' can be obtained with a small investment in extra safety stock. This is highlighted via our numerical example ($\overline{D} = 500,\ \ \sigma_D = 100,\ \ Tp = 2$) by calculating the TNS for eight different values of β.

TABLE 1 *Sample results highlighting the link between bullwhip, inventory and service levels*

β	***Bullwhip***	***NSAmp***	***Bullwhip + NSAmp***	***a*, number of periods coverage required to achieve a 99.5% fill rate**	**Fill rate at constant TNS**
1.667	5.000	3.800	8.800	0.717	99.1%
1.000	1.000	3.000	4.000	**0.622**	**99.5%**
0.618	0.447	3.171	**3.618**	0.643	99.4%
0.500	0.333	3.333	3.666	0.662	99.3%
0.333	0.200	3.800	4.000	0.717	99.1%
0.250	0.143	4.286	4.429	0.773	98.8%
0.167	0.091	5.273	5.364	**0.875**	98.1%
0.100	0.053	7.263	7.316	1.060	96.7%
0.050	0.026	12.256	12.282	1.446	92.8%

From the table, it is clear that we can remove 90% of the order rate variance (i.e. by setting β = 0.167 rather than β = 1) with a quarter of a period extra inventory (0.875 - 0.622 = 0.253), whilst still maintaining a 99.5% fill rate.

The last column of the table shows the fill rate that would result from adopting the smoothing replenishment rule, but maintaining the Target Net Stock at the level required for β = 1. Depending on the profitability of the product (and/or the customer) and the cost of holding inventory, one may elect to 'pay' for smooth replenishments through slightly lower customer service rather than increasing inventory.

Note also that the safety stock required for 99.5% fill-rate at β = 0.333 is the same as for β =1.667, whereas the bullwhip differs by a factor of 25. The "Golden β", 0.61803, minimizes the sum of bullwhip and NSAmp.

In the discussion above, we have presented the bullwhip and customer service as a trade-off situation, in other words as a win-lose situation where one can win on bullwhip and lose on inventory investment (more inventory needed to guarantee the same fill rate). Fortunately, this is not a general conclusion. For certain stochastic demand patterns with Auto Regressive and Moving Average components (ARMA, see Box and Jenkins (1970)) it can be shown that win-win situations do exist. That is, we may win on bullwhip and simultaneously win on inventory levels. Both bullwhip and inventory variability can be reduced simultaneously. We refer the reader to Disney et al. (2004) for a detailed discussion on the win/win opportunities.

5.2. *Underestimating the pipeline inventory*

As noted by Sterman (1989) and confirmed by Croson and Donohue (2002), bullwhip is partly caused by supply chain managers' tendency to under-estimate the pipeline inventory.

We can quantify this statement by mimicking an order-up-to policy with $\beta = \gamma = 1$ and by replacing the WIP term in (4) by $\lambda.WIP_t$

The order quantity is now given by:

$$O_t = \overline{D} + \beta(a\overline{D} - NS_t + Tp\overline{D} - \lambda.WIP_t) \qquad (18)$$

In Figure 7 the results (based on simulation) are given for $\lambda = 0.6$, $\lambda = 1.0$ and $\lambda = 1.3$ (we use the same example as before with $Tp = 3$. Note that overestimating the pipeline stock has no major impact on the bullwhip effect. Under-estimating the pipeline stock, however, has a dramatic impact on the bullwhip effect.

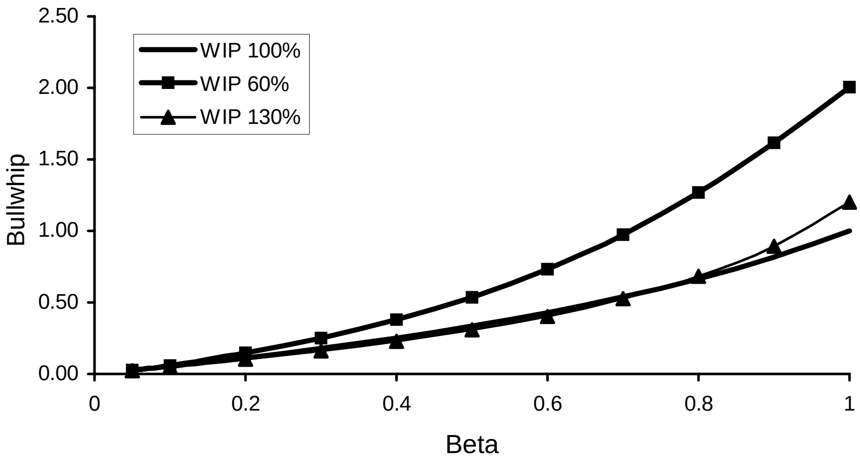

FIGURE 7. The impact of over-and under-estimating pipeline stock on the bullwhip effect

5.3 The impact of forecasting on the bullwhip effect (matched controllers)

In an order-up-to policy, we need to forecast demand over the lead-time and the review period (see expression (1)). As the process is i.i.d., the best possible forecast to use is the average of all previous demands (i.e. $\overline{D}$). Suppose now that decision makers do not know that the underlying demand process is i.i.d. and consequently decide to use a simple forecasting mechanism (e.g. exponential smoothing). Dejonckheere, Disney, Lambrecht and

Towill (2003) showed that applying the replenishment rule (1) (full adjustment and using a forecasting mechanism) will always produce bullwhip ($\sigma^2{}_O / \sigma^2{}_D \geq 1$), irrespective of the demand process. What remains to be answered is what the impact is of forecasting in our smoothing replenishment rule. In other words we implement replenishment rule (5) with matched controllers and we use exponential smoothing (expression (3)) as a forecasting rule. Disney and Towill (2003) have shown (using control theory, and a different notation) that in this case the bullwhip equals:

$$\frac{\beta}{2-\beta}\left(1+\frac{2\alpha(\beta(2-\alpha)+\alpha)}{\beta(\beta(1-\alpha)+\alpha)(2-\alpha)}\right) \qquad (19)$$

The above expression reduces to (8) for $\alpha = 0$ (this is equivalent to using the average demand as a forecaster). For all other values of the smoothing parameter α, the resulting bullwhip effect is always larger than the bullwhip obtained by using $\overline{D}$ as a forecaster.

6. The smoothing rule under stationary demand and unmatched controllers

In section 4 .1 we have set the two feedback controllers β and γ to equal values. This has substantial advantage in terms of simplicity of the bullwhip and net stock variance expressions.

We will now relax this assumption and investigate the dynamics of orders and inventory when the smoothing parameters are set to different values ($\beta <> \gamma$*).* The system then becomes much more complex and furthermore may even become unstable for any demand.

For $\beta <> \gamma$, bullwhip and inventory variance are in general computed through numerical integration. However, we have been able to obtain closed form expressions for the bullwhip and *NSAmp* measures for particular combinations of β, γ and lead times (Tp). The derivation using residue theory is very lengthy, and the resulting expressions are more complex than with $\beta = \gamma$. We have yet to generalize them across lead times.

We will first present results for particularly interesting settings of β , γ and derive some interesting managerial insights.

6.1. The case of $0 \leq \beta \leq 1$ *and* $\gamma = 1$

In this setting, we apply smoothing to the net stock component, but not on the WIP component. We obtain:

$$O_t = \overline{D} + \beta(a\overline{D} - NS_t) + (Tp.\overline{D} - WIP_t) \qquad (20)$$

This policy was introduced by Magee (1956), who demonstrated that the bullwhip resulting from a replenishment rule with $\gamma = 1$ is equal to our result obtained in (8) for the case of $\beta = \gamma$. Why would we want to introduce a more complex rule when a simpler setting yields the same result? The answer lies in the variance of inventory, which is larger than in the case of $\beta = \gamma$. The net stock amplification is now given by:

$$NSAmp_{\gamma=1} = \frac{Tp + 1}{(2-\beta)\beta} \qquad (21)$$

This results in a larger net stock variance compared to the matched controllers case (for $0 \leq \beta \leq 2$) , see Figure 8.

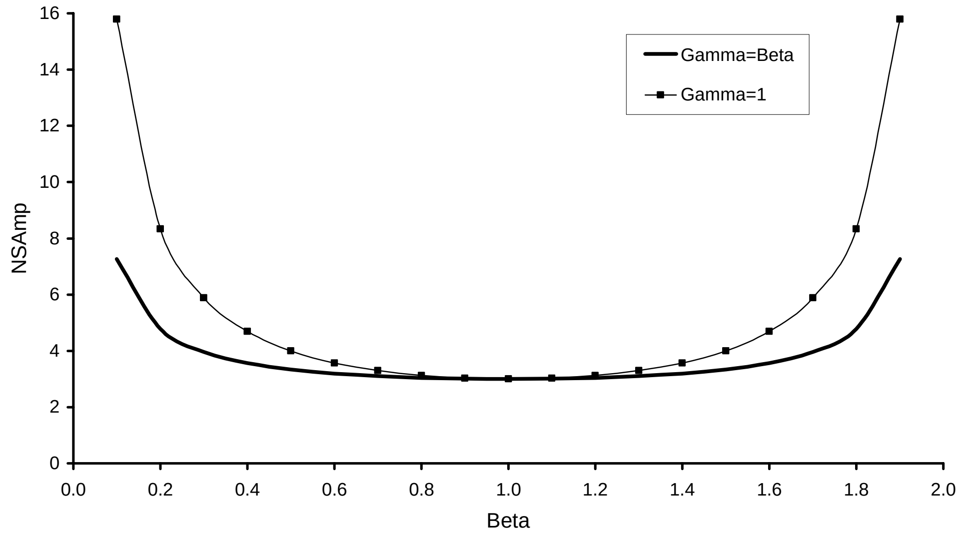

Figure 8: NSAmp for $\beta = \gamma$ and $\gamma = 1$, *Tp*=2

6.2. The case of $\beta = 1$ *and* $\gamma \neq 1$

In this setting, the order quantity is given by:

$$O_t = \overline{D} + (a\overline{D} - NS_t) + \gamma(Tp.\overline{D} - WIP_t) \qquad (22)$$

Figure 9 shows the bullwhip for lead times Tp ranging from 0 to 5 periods, combined with $\gamma = 0.5$ and $\gamma = 1.4$. Recall that bullwhip equals unity if $\beta = \gamma = 1$.

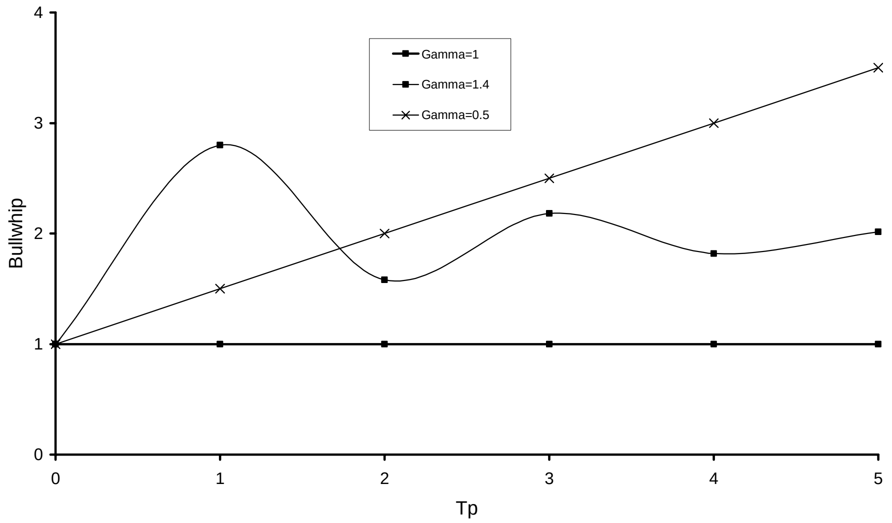

Figure 9: Bullwhip for under/over-reactions to changes in WIP

6.3. Minimizing bullwhip by tuning β<>γ

The next question we want to investigate is whether careful tuning of the feedback controllers can lead to less bullwhip compared to the matched controller case.

We have derived closed forms for the bullwhip expression with $\beta <> \gamma$ for a specified lead time (Tp) up to 5 periods, see Table 2.

TABLE 2 *Bullwhip expressions with unmatched controllers*

Tp	**Bullwhip**
0	$\frac{\beta}{2-\beta}$
1	$\frac{\beta}{2(1+\beta-\gamma)-\beta}\cdot\frac{1+\beta-\gamma}{1-\beta+\gamma}$
2	$\frac{\beta}{2-\beta}\cdot\frac{1+\beta-\gamma-\beta(\beta-\gamma)}{1-\beta+\gamma-(\beta-2\gamma)(\beta-\gamma)}$
3	$\frac{\beta}{2(1+\beta-\gamma)-\beta}\cdot\frac{\beta\left(2-\beta-(\beta-2\gamma)(\beta-\gamma)\right)+(1-\gamma)^2}{\left(1+\beta^3+\left(2-3\gamma\right)\gamma-\beta^2\left(1+3\gamma\right)+2\beta\left(\gamma\left(2+\gamma\right)-1\right)\right)}$
4	$\frac{\beta}{2-\beta}\cdot\frac{\beta\left(2-\beta\right)\left(1-\beta+\gamma-(\beta-2\gamma)(\beta-\gamma)\right)+(1-\gamma)^2}{\left(\beta^4+\beta^3\left(1-5\gamma\right)+\left(\gamma-1\right)^2\left(1+4\gamma\right)-2\beta\left(\gamma-1\right)\left(2\gamma\left(2+\gamma\right)-1\right)+\beta^2\left(\gamma\left(8\gamma-1\right)-3\right)\right)}$
5	$\frac{\beta}{2(1+\beta-\gamma)-\beta}\cdot\frac{\begin{pmatrix}-\beta^5+\left(\gamma-1\right)^3-3\beta\left(\gamma-1\right)^2\left(1+2\gamma\right)+\\ \beta^2\left(\gamma-1\right)\left(3+\gamma\right)\left(4\gamma-1\right)+\beta^4\left(5\gamma-1\right)+\beta^3\left(4-8\gamma^2\right)\end{pmatrix}}{\begin{pmatrix}\beta^5-\beta^4\left(1+5\gamma\right)-\left(\gamma-1\right)^2\left(1+5\gamma\right)+\\ \beta^3\left(8\gamma\left(1+\gamma\right)-4\right)+\beta\left(\gamma-1\right)\left(\gamma\left(9+10\gamma\right)-3\right)+\beta^2\left(3+\gamma\left(10-\gamma\left(17+4\gamma\right)\right)\right)\end{pmatrix}}$

The complexity of the bullwhip formula increases with the lead-time and there appears to be an interesting interaction between odd and even lead-times that we have yet to fully understand.

Let's consider the case of $Tp = 1$ for illustrative purposes. Figure 10 shows the value of γ that minimizes the bullwhip (on the right hand Y-axis), for a given β and the resulting bullwhip (on the left hand Y-axis). The dotted line is the bullwhip assuming matched controllers.

It is clear that careful tuning of γ will further reduce the bullwhip compared to the matched controllers. It is interesting to note that the bullwhip minimizing γ is always larger than β. Simulation shows that as Tp approaches infinity, bullwhip is minimized by setting $\gamma = 0.5 + 0.5\beta$

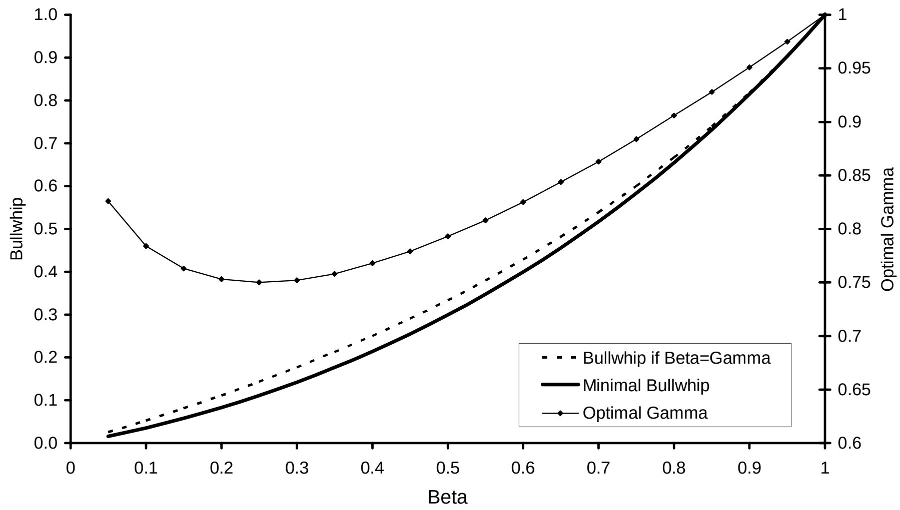

FIGURE 10: Minimizing bullwhip by tuning $\beta \neq \gamma$

Analysis reveals that although careful tuning of the feedback controllers can reduce bullwhip, this comes at the expense of increased net stock variance. Again, we have yet to identify the closed form *NSAmp* expression for the general lead-time case, although we have obtained the ratios for specific lead-times. Table 3 details the *NSAmp* expression for the case of lead-times up to 5 periods. It can also be shown via differentiation and use of the Hessian determinant that even for the unmatched controller case, the sum of bullwhip and *NSAmp* (see Figure 6), is minimized at $\beta = \gamma = 0.618$. The golden ratio proves to be a particularly strong result and if we are interested in the sum of the bullwhip effect and the net stock amplification, then the matched controller case ($\beta = \gamma$) dominates the unmatched controller case.

TABLE 3: *Net Stock variance amplification with unmatched controllers*

Tp	*NSAmp*
0	$\dfrac{1}{\beta(2-\beta)}$
1	$\dfrac{(1-\gamma)(1+\gamma)^2+\beta(1+\gamma^2)}{\beta(2-\beta-(\beta-2\gamma)(\beta-\gamma))}$
2	$\dfrac{1+\beta-\beta^2+(3-\beta)\gamma+2\gamma^2(2-\beta)(\beta-\gamma)}{(\beta-2)\beta(\beta+\beta^2-3\beta\gamma-(1-\gamma)(1+2\gamma))}$
3	$\dfrac{(\gamma-1)^2(1+3\gamma)^2-\beta^3(1+3\gamma^2)-2\beta(\gamma-1)(1+\gamma+7\gamma^2+3\gamma^3)+\beta^2(\gamma-1+\gamma^2+9\gamma^3)}{\beta(2+\beta-2\gamma)(1+\beta^3+(2-3\gamma)\gamma-\beta^2(1+3\gamma)+2\beta(\gamma(2+\gamma)-1))}$
4	$\dfrac{\begin{pmatrix}1+(\beta-2)\beta(\beta-1+\beta^2)+6\gamma-\beta(2+(\beta-1)\beta)\gamma+\\(1+2(\beta-2)\beta(2+\beta)(2\beta-3))\gamma^2-6(4+\beta^2(2\beta-5))\gamma^3+8(\beta-2)(\beta-1)\gamma^4\end{pmatrix}}{\left(\beta(2-\beta)\begin{pmatrix}\beta^4+\beta^3(1-5\gamma)+(\gamma-1)^2(1+4\gamma)-\\2\beta(\gamma-1)(2\gamma(2+\gamma)-1)+\beta^2(\gamma(8\gamma-1)-3)\end{pmatrix}\right)}$
5	$\dfrac{\begin{pmatrix}(\gamma-1)^3(1+5\gamma)^2-\beta^5(1+5\gamma^2)-\beta(\gamma-1)^2(3+\gamma(6+55\gamma+50\gamma^2))-\\4\beta^3(\gamma^2(2\gamma(2+5\gamma)-7)-1)+\beta^4(\gamma(3+\gamma(25\gamma-3))-1)+\\\beta^2(\gamma-1)(\gamma(1+\gamma(\gamma(89+20\gamma)-11))-3)\end{pmatrix}}{\left(\beta(2+\beta-2\gamma)\begin{pmatrix}\beta^5-\beta^4(1+5\gamma)-(\gamma-1)^2(1+5\gamma)+\beta^3(8\gamma(1+\gamma)-4)+\\\beta(\gamma-1)(\gamma(9+10\gamma)-3)+\beta^2(3+\gamma(10-\gamma(17+4\gamma)))\end{pmatrix}\right)}$

At this point, we like to point out the analogy between a supply chain manager's choice of replenishment policies and a financial analyst's selection of investment opportunities. Both managers optimize a portfolio from a risk/return perspective. The supply chain manager's return is the benefit from smooth replenishments; the risk is in the increased inventory variability. This allows us to draw an "efficient frontier" curve (see Figure 11), which we can interpret as follows: for a particular level of bullwhip (return), there is no set of β and γ values that yields a lower *NSAmp* (risk) as indicated on the curve, and vice versa.

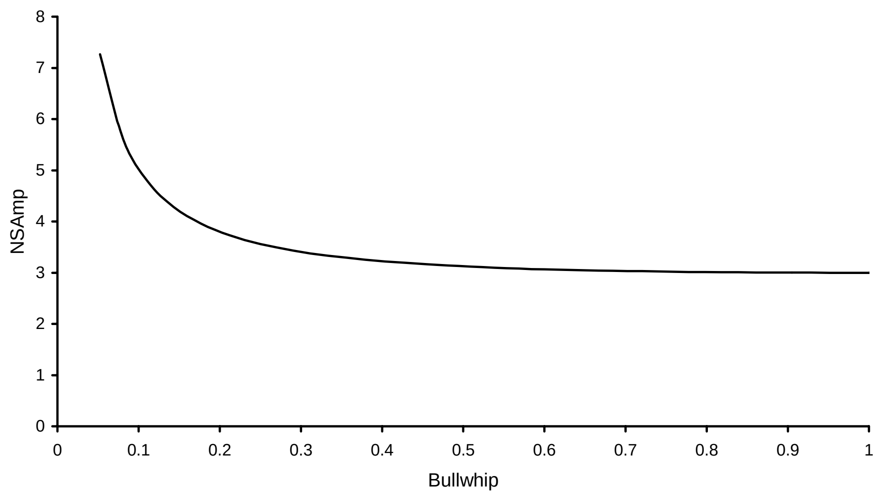

Figure 11: The efficient frontier curve, for $Tp = 2$

7. Conclusions

The bullwhip effect has been studied by many authors in recent years. Currently, taming the bullwhip seems to be a dominating operations strategy. Indeed, the bullwhip effect has a number of highly undesirable cost implications. In this paper we show that much conventional bullwhip reduction is focusing only on one side of the coin. In developing a replenishment rule, the decision maker has to consider two factors: firstly, the investment in inventories and the customer service which is related to net stock variability and secondly, the order variability which is related to the bullwhip effect. Previously, both aspects have been studied separately. In this paper we combine both factors into a single model. A smoothing replenishment rule is developed, capable of balancing order variability and net stock variability. We assume an independently and identically distributed demand process. The authors have also developed a replenishment rule for more general demand processes (see Disney, Farasyn, Lambrecht, Towill and Van de Velde (2004)).

The main conclusion herein is that bullwhip reduction (dampening order variability) comes at a price. More specifically, in order to guarantee the same fill rate, more investment in safety stock is required. A lot of dampening, however, can be obtained by a small increase in safety stock. The issue of matched or unmatched feedback controllers is also uniquely raised in this paper. The matched controller's case dominates the unmatched controller case if we are interested in the combined effect of bullwhip and net stock amplification. The "golden β" turns out to be key value of the smoothing parameter.

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